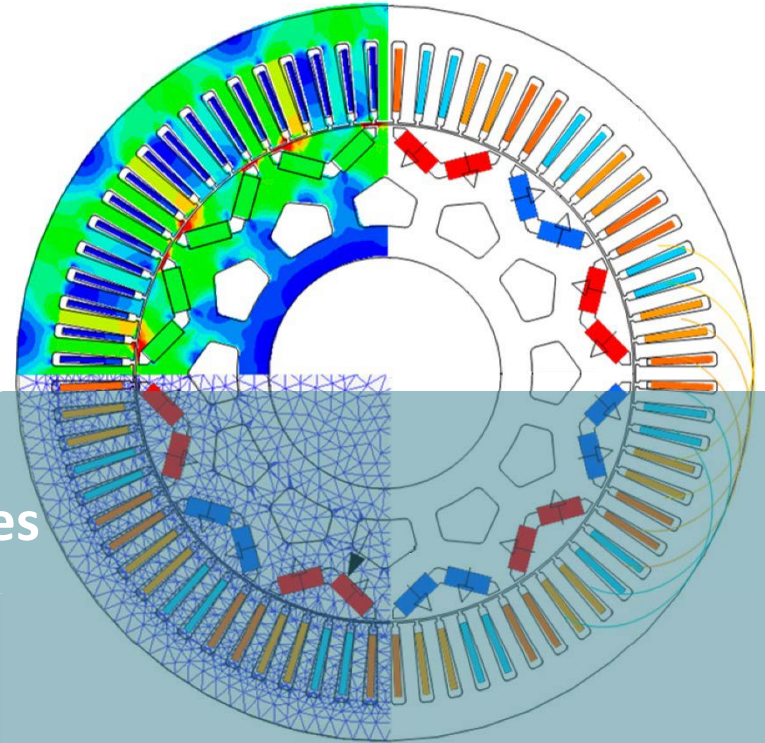
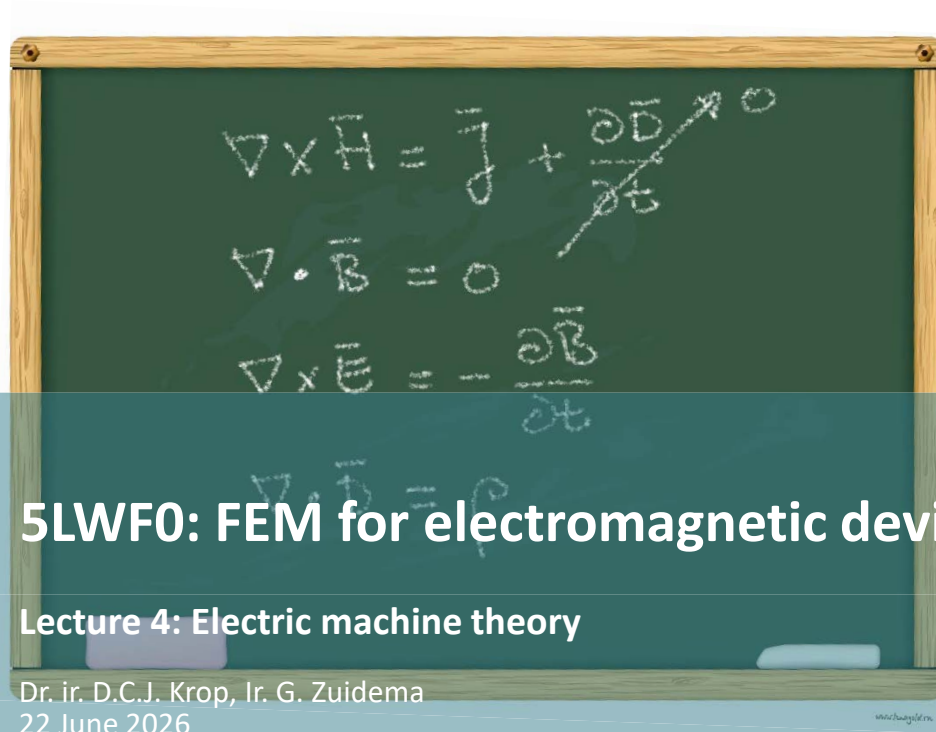




Outline and topics of the previous lecture

Interactive demonstration on performing calculations in FLUX2D

- Model reduction by exploiting geometric and physical symmetries and/or periodicities

EMF and inductance calculations

- Flux linkage calculations in FLUX2D
- Inductance and EMF calculations from a magnetostatic model
 - Exporting flux linkage data to Matlab and perform postprocessing
- Inductance and EMF calculations from a transient model
 - Direct EMF-calculation

Outline for the lecture of week 4

Recap of 3-phase synchronous machine theory

- Commutation of phase-currents
- Torque generation
- AC vs. DC operation
- Machine constant

Torque ripples

- Cogging
- Reluctance torque
- Harmonic content in excitation

Winding configurations

3 Phase synchronous machines



Creation of a rotating magnetic stator-field

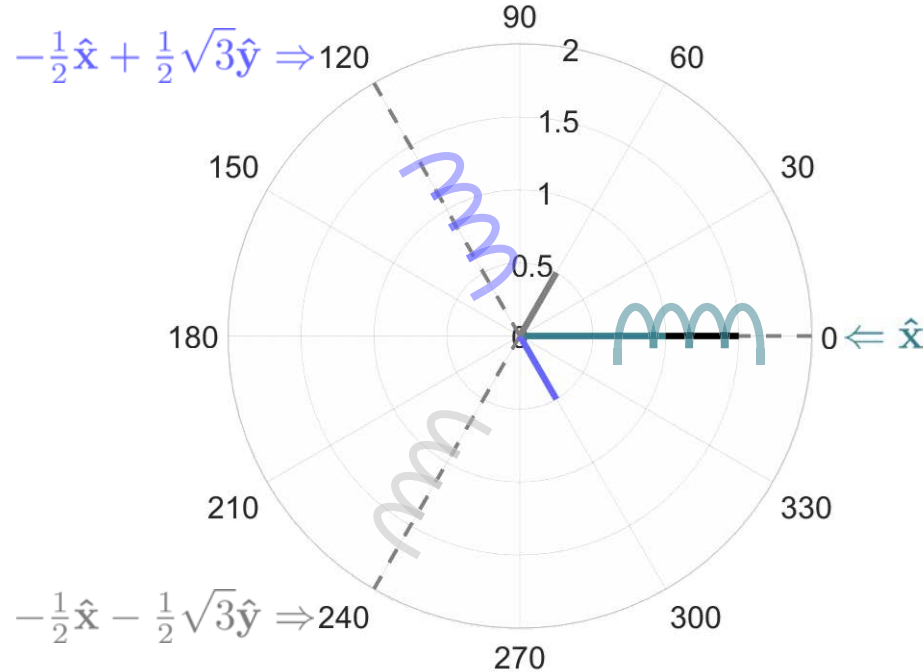
Three phase system: 120° phase shift in **both** space and time

$$\left. \begin{aligned} \vec{B}_A &= \hat{B} \cos(\omega t) \hat{\mathbf{x}} \\ \vec{B}_B &= \hat{B} \cos(\omega t - \tfrac{2}{3}\pi) \left[-\tfrac{1}{2}\hat{\mathbf{x}} + \tfrac{1}{2}\sqrt{3}\hat{\mathbf{y}} \right] \\ \vec{B}_C &= \hat{B} \cos(\omega t + \tfrac{2}{3}\pi) \left[-\tfrac{1}{2}\hat{\mathbf{x}} - \tfrac{1}{2}\sqrt{3}\hat{\mathbf{y}} \right] \end{aligned} \right\} \Rightarrow \vec{B} = \vec{B}_A + \vec{B}_B + \vec{B}_C$$

$$\begin{aligned} B_x &= \hat{B} \cos(\omega t) - \frac{1}{2}\hat{B} \overbrace{\left[\cos(\omega t - \tfrac{2}{3}\pi) + \cos(\omega t + \tfrac{2}{3}\pi) \right]}^{2 \cos\left(\frac{2}{3}\pi\right) \cos(\omega t)} = \frac{3}{2}\hat{B} \cos(\omega t) \\ B_y &= \frac{1}{2}\sqrt{3}\hat{B} \overbrace{\left[\cos(\omega t - \tfrac{2}{3}\pi) - \cos(\omega t + \tfrac{2}{3}\pi) \right]}^{2 \sin\left(\frac{2}{3}\pi\right) \sin(\omega t)} = \frac{3}{2}\hat{B} \sin(\omega t) \end{aligned}$$

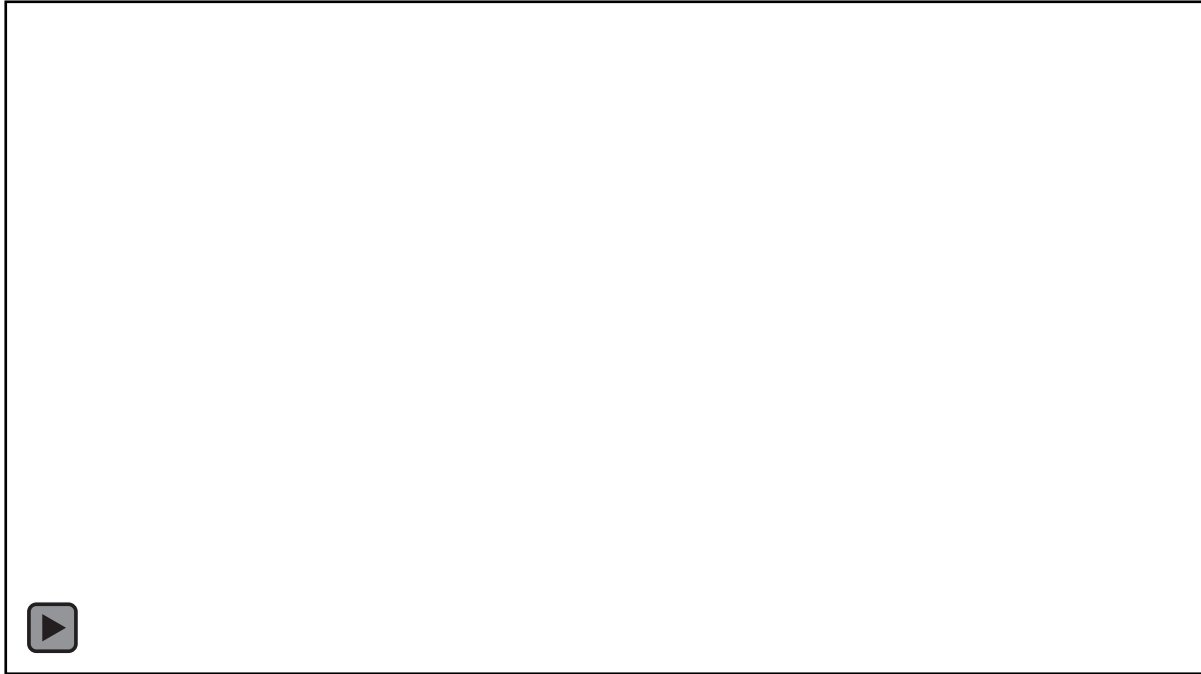
Creation of a rotating magnetic field stator-field

Three phase system: 120° phase shift in **both** space and time



Creation of a rotating magnetic field stator-field

Three phase system: 120° phase shift in **both** space and time



How many poles does this machine have?

4 poles

or

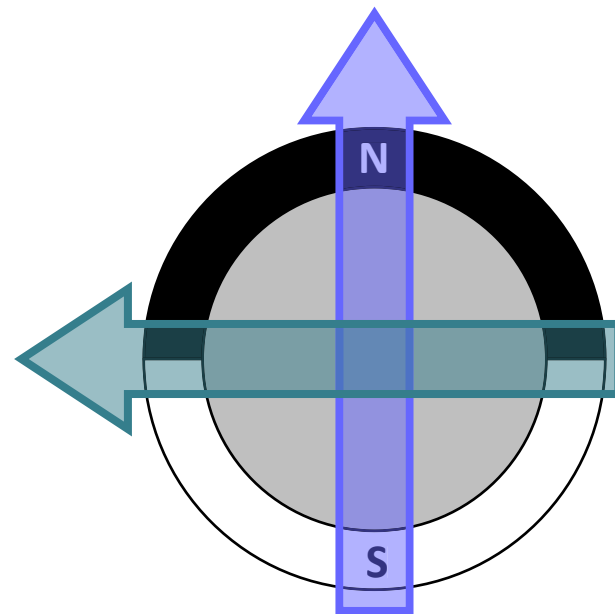
2 pole-pairs (north-south pairs)

Creation of the rotor-field

In 5LWF0 only rotors magnetized by permanent magnets

Conventions:

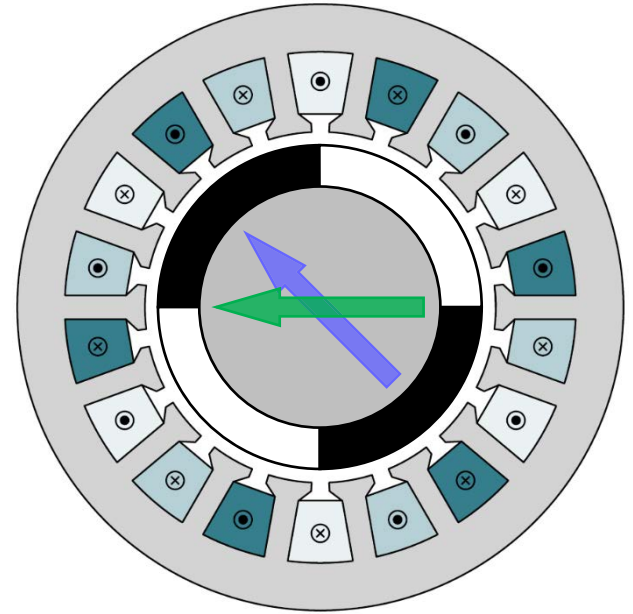
- The **direct d -axis** of the rotor is *parallel* to the *positive* direction of magnetization of the **rotor** (and not necessarily the magnet! *Why?*)
- The **quadrature q -axis** of the rotor is *perpendicular* to the **positive** direction of magnetization of the **rotor** (generally *leading* the **d -axis** by 90 electrical degrees)



Creation of the rotor-field

Three phase system: 120° phase shift in **both** space and time

- The **direct d -axis** generates a torque such to seek alignment with the **rotating stator field**
- Highest torque is obtained when the **d -axis** is 90° (electrical degrees!) displaced in space with respect to the **rotating stator field**
 - i.e. **stator field** aligned with the **q -axis**!
- The **rotating stator field** causes armature reaction
 - Commutation angle has to be adjusted accordingly



Instantaneous torque production

EMF of the k^{th} phase:
$$\mathcal{E}_k(\theta) = -\frac{d\Psi_k(\theta)}{dt} = -\frac{d\Psi_k(\theta)}{d\theta} \frac{d\theta}{dt} = -\omega_m \frac{d\Psi_k(\theta)}{d\theta}$$

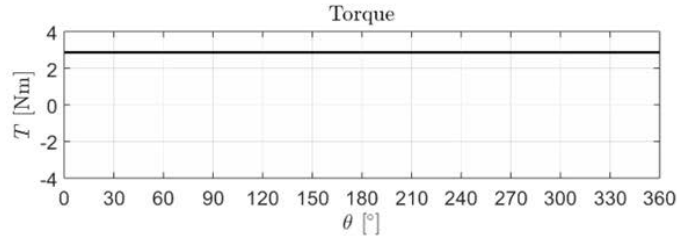
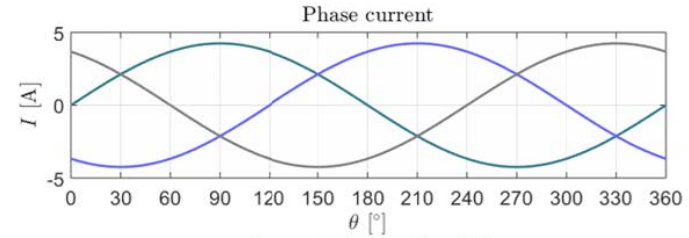
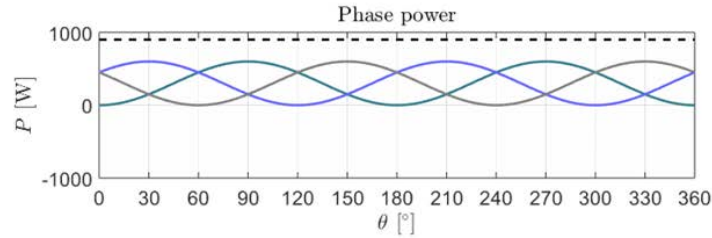
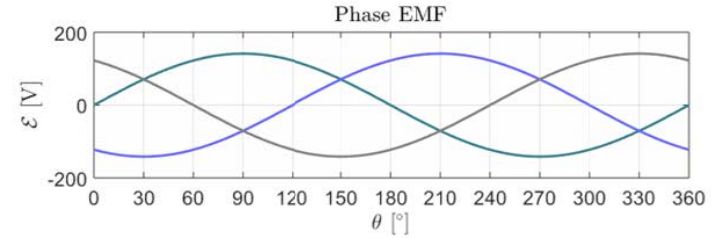
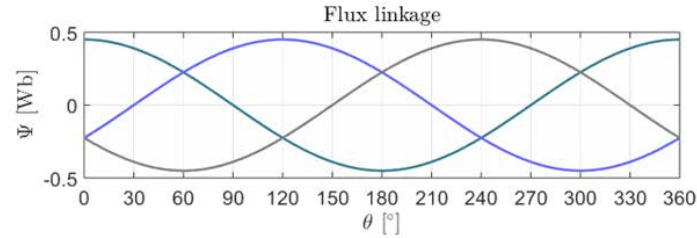
Electrical power of the k^{th} phase:
$$P_{e,k}(\theta) = \mathcal{E}_k(\theta)i_k(\theta) = -\omega_m i_k \frac{d\Psi_k(\theta)}{d\theta}$$

Mechanical power per phase:
$$P_{m,k}(\theta) = \omega_m T_k(\theta)$$

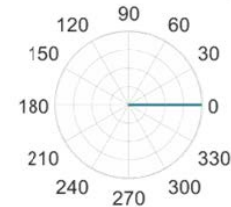
Power balance $P_e = P_m$:

$$T_{\text{em}}(\theta) = \frac{1}{\omega_m} \sum_{k=1}^K \mathcal{E}_k(\theta)i_k(\theta) = -\sum_{k=1}^K \frac{d\Psi_k(\theta)}{d\theta} i_k$$

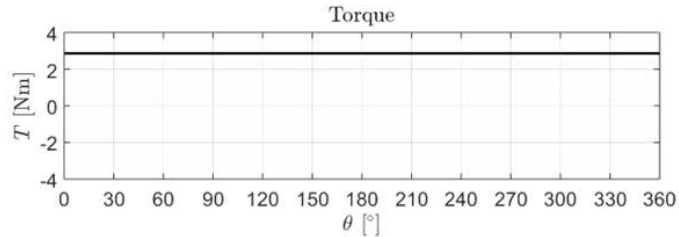
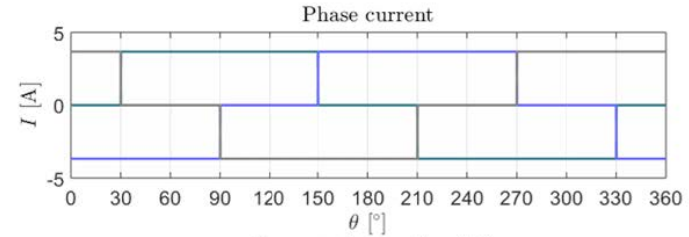
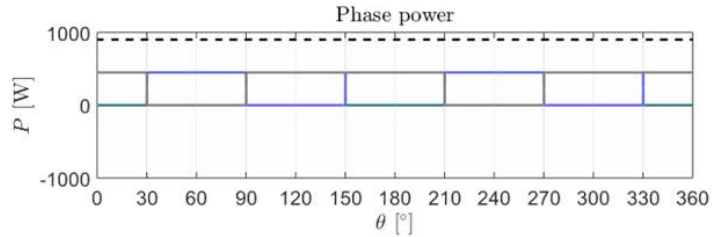
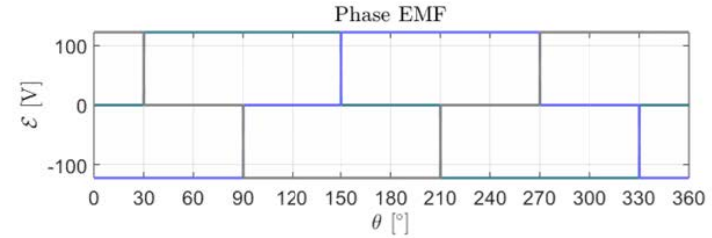
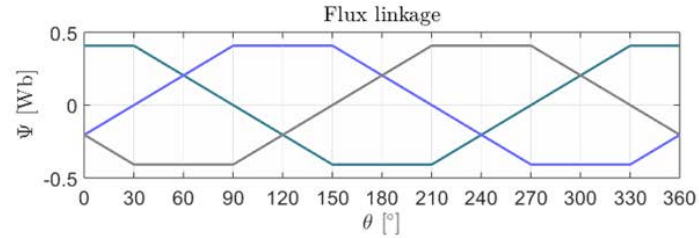
3-Phase AC operation



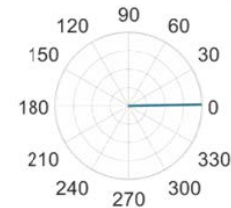
Commutation angle: ϕ [°]



3-Phase DC operation



Commutation angle: ϕ [°]



AC/DC operation: phase shift between EMF and current

AC operation

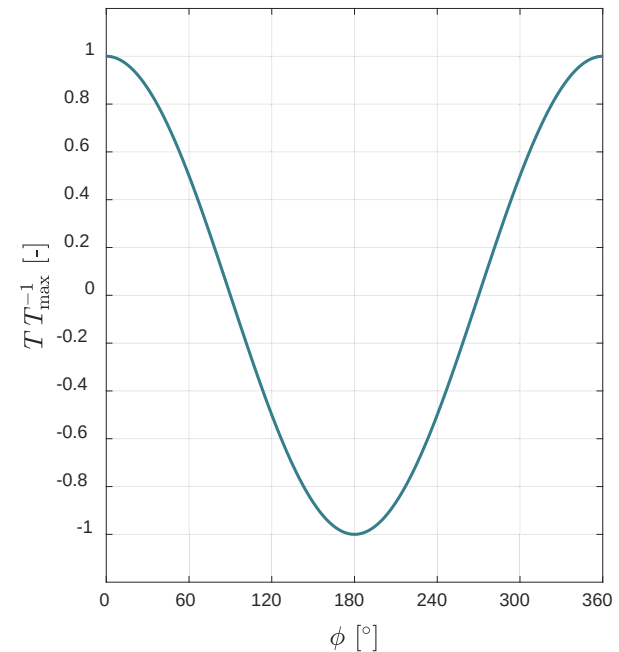
- Torque still constant for AC operation
- Smooth 'scaling' of the torque with respect to the phase shift for AC operation
- Phase shifts allow field weakening operation

DC operations

- Large torque ripple for phase shifts not equal to an integer multiple of 60°
- Field weakening operation not applied in DC operation

Is $\cos(\phi)$ the power factor of the machine?

$$T_{\text{em}}(\theta) = \frac{3}{\omega_m} \mathcal{E}_{\text{rms}} I_{\text{rms}} \cos(\phi)$$



Machine constant(s)

Power balance @ steady-state conditions (proportionality between **EMF** and **speed**)

$$N_{\text{ph}} \mathcal{E} I = \omega_m T \Rightarrow$$
$$k_{\mathcal{E}} = \frac{\mathcal{E}}{\omega_m} = \frac{T}{N_{\text{ph}} I} \quad [\text{V s rad}^{-1}]$$

or alternatively (proportionality between **current** and **torque**):

$$k_T = \frac{T}{I} = \frac{N_{\text{ph}} \mathcal{E}}{\omega_m} = N_{\text{ph}} k_{\mathcal{E}} \quad [\text{N m A}^{-1}]$$

Ideally, the **EMF**- and **torque-constant** are related as:

$$k_T = N_{\text{ph}} k_{\mathcal{E}}$$

Should not the constants then have the same SI-unit?

Dimensional analysis:

- Unit of angular velocity:

$$[\text{rad s}^{-1}] \Leftrightarrow [\text{s}^{-1}]$$

- Hence

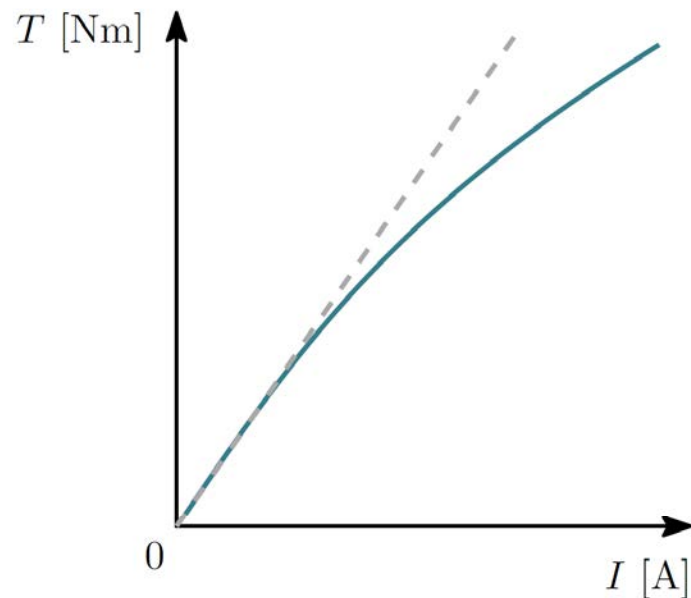
$$\begin{aligned} [\text{V s}] &= [\text{J C}^{-1} \text{s}] \\ &= [\text{J A}^{-1}] \\ &= [\text{N m A}^{-1}] \end{aligned}$$

Machine constant(s)

The **EMF-constant** can only be determined under steady-state, no-load conditions ($I=0$); whereas the **torque-constant** requires current to flow ($I \neq 0$)

Linearity of the machine 'constant' is lost due to

- Saturation (non-linear, soft-magnetic material)
- Armature reaction
- Higher harmonic content in
 - EMF
 - Phase current



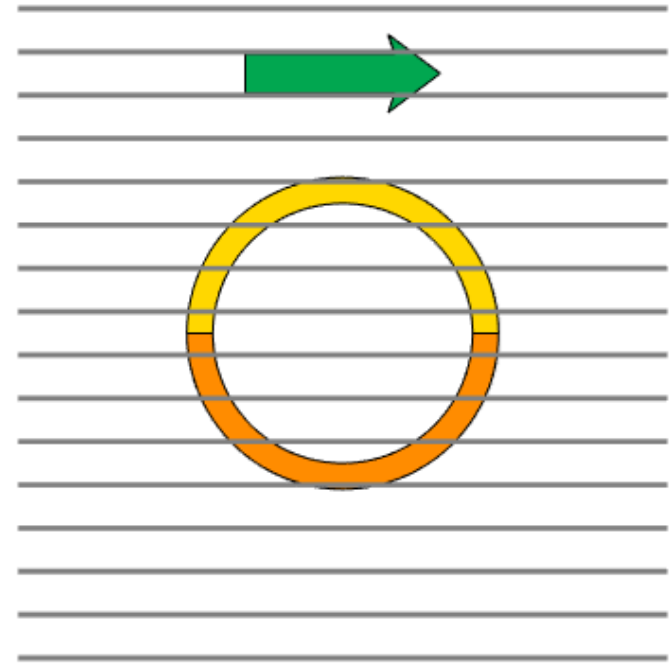
Armature reaction

Armature reaction: distortion of the **stator field** due to the orthogonal **rotor field**

- Adjustment of commutation angle
- Variation in machine constant

Link to DC-machines

- Use of inter-poles and compensation windings to prevent sparking brushes (miscommutation)



Flux linkage in multi-source problems: generic system

The time-varying flux linkage of coil k in a **generic** K -coil system with **permanent magnets**, **movement** and K **mutually independent**, but **position controlled currents**

$$\Psi_k \left(\theta_m(t), \vec{i}(\theta_m(t)) \right), \quad \vec{i}(t) = [i_1(\theta_m(t)), i_2(\theta_m(t)), \dots, i_k(\theta_m(t)), \dots, i_K(\theta_m(t))]$$

Voltage of coil k @ $t=\tau$ due to the simultaneous changes in **movement** and **currents**

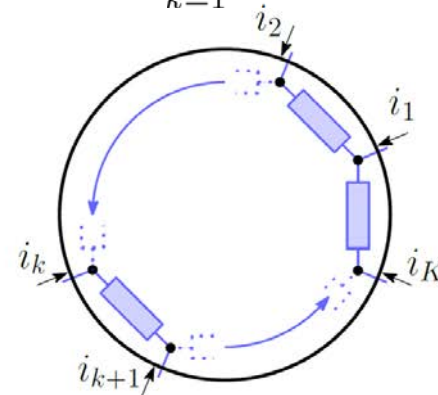
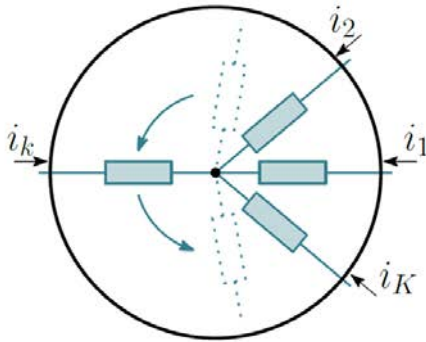
$$\begin{aligned} V_k(\tau) &\propto \frac{d\Psi_k}{dt} = \omega_m \left(\frac{\partial \Psi_k}{\partial \theta_m} + (\nabla_i \Psi_k) \cdot \frac{d\vec{i}}{d\theta_m} \right) \bigg|_{\vec{i}=\vec{i}(\tau), \theta_m=\theta_m(\tau)} \\ &\propto \omega_m \hat{\mathcal{E}}_k + \omega_m [L_{k1}, \dots, L_{kk}, \dots, L_{kK}] \cdot \left[\frac{\partial i_1}{\partial \theta_m}, \dots, \frac{\partial i_k}{\partial \theta_m}, \dots, \frac{\partial i_K}{\partial \theta_m} \right] \end{aligned}$$

EMF (Incremental) inductance vector 'Commutation' vector

Flux linkage in multi-source problems: multi-phase system

For ***K*-phase** system only *K*-1 independent currents

- Kirchhoff's current law in **Wye ('star')** or **Delta ('ring')** connections: $\sum_{k=1}^K i_k = 0 \Rightarrow i_K = - \sum_{k=1}^{K-1} i_k$

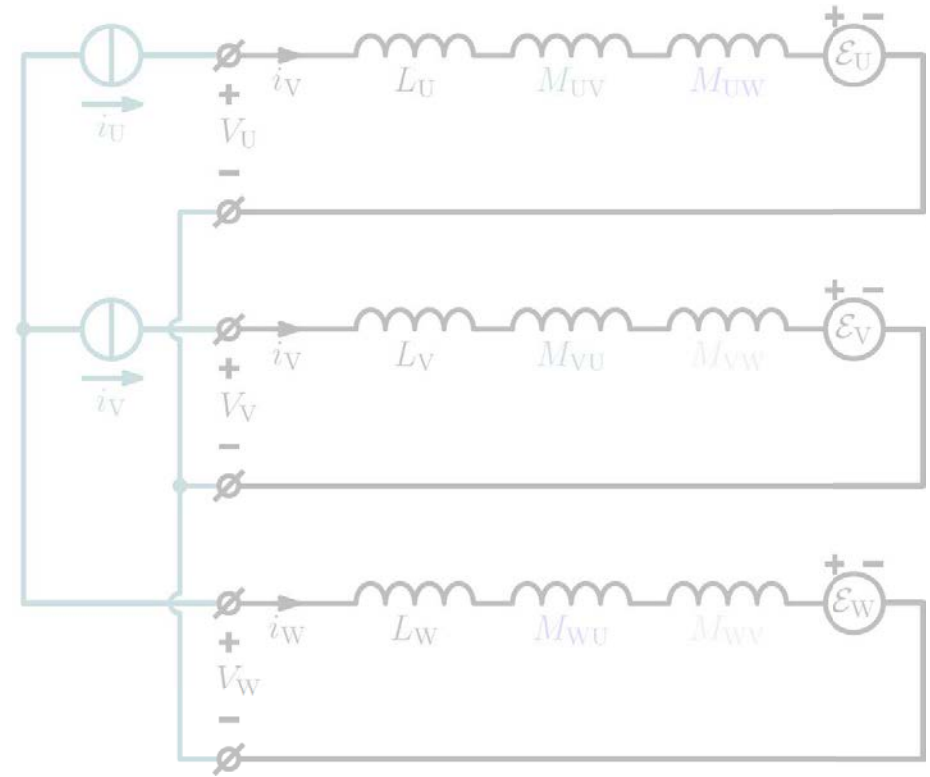
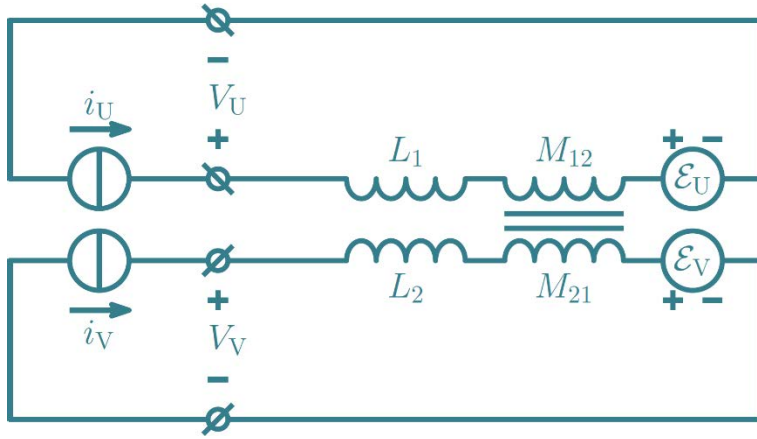


$$\frac{V_k}{\omega_m} \propto \hat{\mathcal{E}}_k + [L_{k1}, \dots, L_{kk}, \dots, L_{k(K-1)}] \cdot \left[\frac{\partial i_1}{\partial \theta_m}, \dots, \frac{\partial i_k}{\partial \theta_m}, \dots, \frac{\partial i_{K-1}}{\partial \theta_m} \right]$$

Flux linkage in multi-source problems: 3-phase system

For **3-phase** system (UVW) only **2** independent currents

$$\frac{V_U}{\omega_m} \propto \hat{\mathcal{E}}_U + L_1 \frac{\partial i_U}{\partial \theta_m} + M_{12} \frac{\partial i_V}{\partial \theta_m}$$



Flux linkage in multi-source problems: 3-phase system

- a) Why are L_1 and M_{12} **not** equal to the actual self- and mutual phase-inductances?
- L_1 and M_{12} are calculated based on an already realized coil connection, e.g. Wye. These values already include the flux originating from the third **dependent** current. They are effective values experienced by two phases as the result of all three!
- b) Is the EMF equal to the no-load EMF?
- Only when all phase currents are zero. In general, the EMF is a function of the phase currents. This is logical as phase currents pre-bias the soft-magnetic material, which affects the flux distribution from the permanent magnets due to saturation effects.
- c) Why **is** the EMF equal to the actual phase-EMF, contrary to the inductances?
- The EMF is the result of changes in the flux linkage due to changes in position. There is no dependency between the position and any of the currents as there is for the currents due to the connection. The EMF is affected by the value of currents, but not to changes in the currents. See b)